PANIC 05 , Santa Fe, October 24 2005

Factorization in exclusive semileptonic radiative B decays

Vincenzo Cirigliano

Based on hep-ph/0508095 with Dan Pirjol

Motivation – why look at B → π lν γ ?

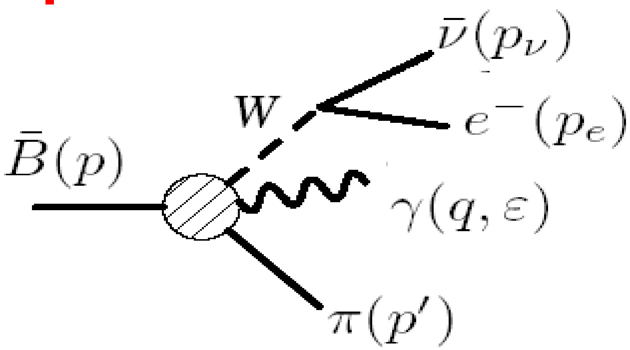
- Radiative effects [virtual and real] will soon become an issue at B factories for several channels ➔
  need theoretical input to test/improve MC simulations

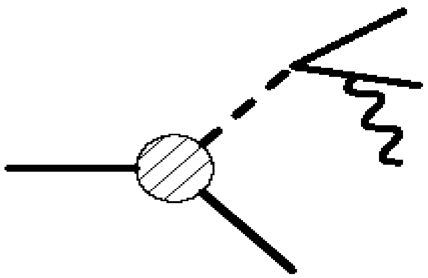
  www.slac.stanford.edu/BFROOT/www/Public/Organization/2005/workshops/radcorr2005/index.html

- In kinematical region of hard γ and soft π we can make predictions combining SCET + ChPT: interesting on its own !

- In this talk:
  - ➔ present factorization formula – sketch proof
  - ➔ phenomenology: BR[cuts], distributions in E_γ and $\theta_{e\gamma}$ (including comparison with simplified MC treatments)

Amplitude decomposition – form factors





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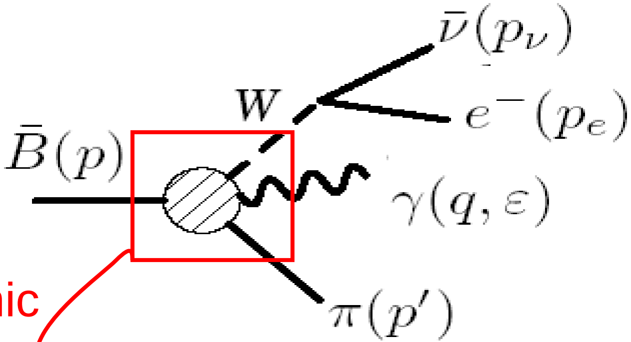

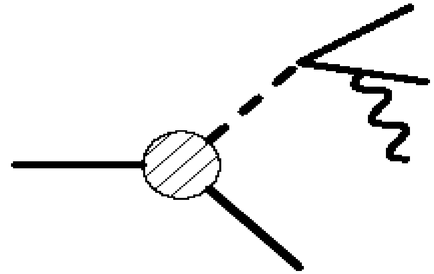

Hadronic tensor

$$T^J_{\mu\nu} = i\int d^4x e^{iq\cdot x}\langle \pi(p')|\mathrm{T}\left\{j^{\mathrm{em}}_\mu(x), J_\nu(0)\right\}|\bar{B}(p)\rangle$$

$\boxed{\bar{u}\gamma_\nu b}$ $\boxed{\bar{u}\gamma_\nu\gamma_5 b}$

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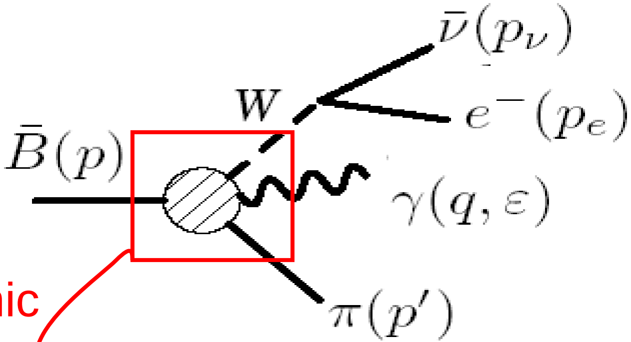

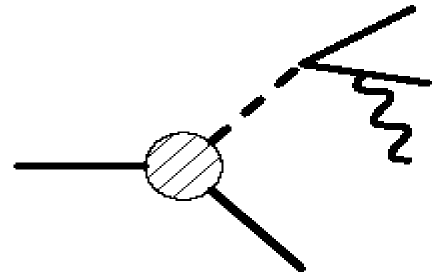

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Form Factors ($W = p_e + p_\nu$)

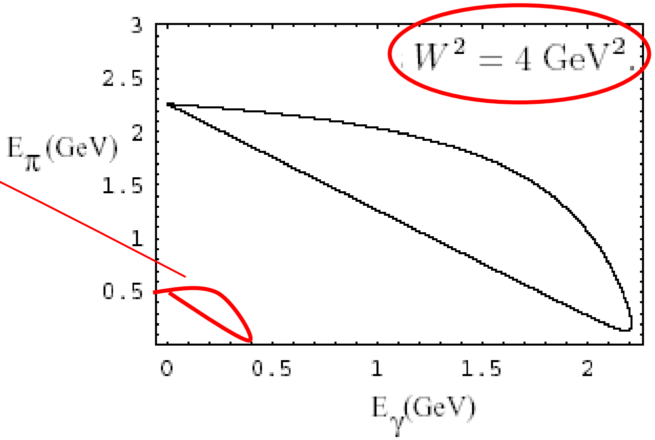
$$\boxed{\begin{aligned}\varepsilon^{*\mu}T^V_{\mu\nu} &= V_1\left(\varepsilon^*_\nu - \frac{\varepsilon^*\cdot W}{q\cdot W}q_\nu\right) + \left(p'\cdot\varepsilon^* - \frac{(p'\cdot q)(\varepsilon^*\cdot W)}{q\cdot W}\right)(V_2 q_\nu + V_3 W_\nu + V_4 p'_\nu) - \frac{\varepsilon^*\cdot W}{q\cdot W}\langle\pi(p')|\bar{u}\gamma_\nu b|\bar{B}(p)\rangle \\ \varepsilon^{*\mu}T^A_{\mu\nu} &= i\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}(A_1 p'^\rho q^\sigma + A_2 q^\rho W^\sigma) + i\epsilon_{\mu\lambda\rho\sigma}\varepsilon^{*\mu}p'^\lambda q^\rho W^\sigma(A_3 W_\nu + A_4 p'_\nu)\end{aligned}}$$

$V_{1,..,4}$ and $A_{1,..,4}$ are functions of 3 variables: use W^2, E_π , E_γ

What do we know about the amplitude?

- Large W^2 ➔ E_π , E_γ << $\Lambda_{\chi SB}$ → Heavy Hadron ChPT

$$\boxed{m_e^2 \leq W^2 \leq (M_B - M_\pi)^2}$$



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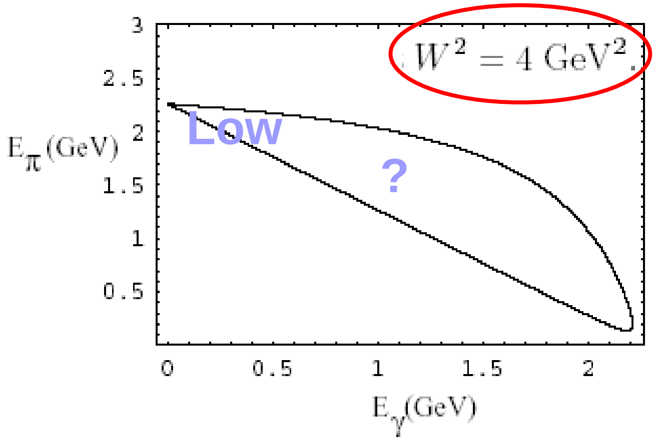

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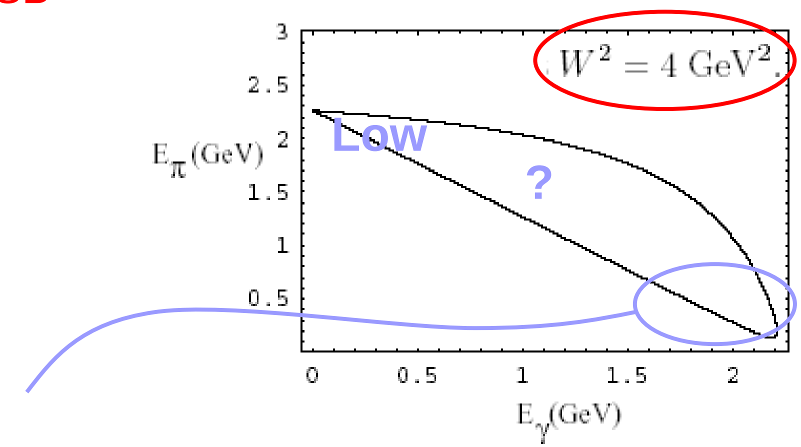


Hierarchy of scales leads to factorized form of amplitude

$$Q \sim \{E_\gamma, m_b\} \quad \gg \quad \Lambda \sim \{E_\pi, \Lambda_{\mathrm{QCD}}\}$$

$$\boxed{A(B \to \pi \ell \nu \gamma) = \underbrace{H \cdot J}_{\text{Perturbatively calculable}} \otimes \underbrace{S(B \to \pi)}_{\text{“Soft” matrix element of bi-local operator}} + O(\Lambda/Q)}$$

Perturbatively calculable

“Soft” matrix element of bi-local operator
Use HHChPT to relate it to B-meson LCDA

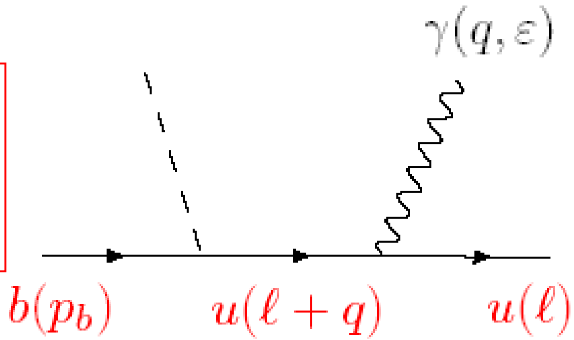
Factorization I

To leading order in α_s and Λ/Q , $B \to \pi\ell\nu\gamma$ is mediated by

$q = n E_\gamma$

$E_\gamma \gg \Lambda$

$\ell \sim \Lambda$



$$(q+\ell)^2 \sim 2E_\gamma\,(n\cdot\ell) \sim Q\,\Lambda \gg \Lambda^2$$

➔ Integrate out hard-collinear light quark

$$\langle T_{\mu\nu}\rangle \sim i e_u\,\bar{u}(\ell)\,\frac{\gamma_\mu \not{n}\,\gamma_\nu}{2n\cdot\ell + i\epsilon}\,b(p_b)$$

Dependence on light-cone projection of ℓ ➔
EFT involves bi-local light-cone operators

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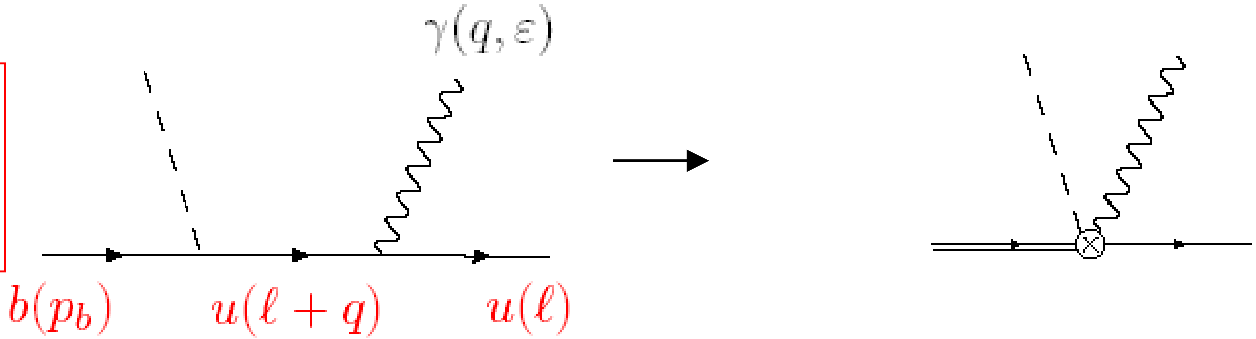

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Dependence on light-cone projection of ℓ ➔
EFT involves bi-local light-cone operators

$$\int d^4x\, e^{iq.x}\, T\{(\bar{q}\gamma_\nu P_L b)(0)\,, j_\mu^{\mathrm{e.m.}}(x)\} \longrightarrow \int dt\, [e_u \theta(t)]\, \boxed{\bar{u}(nt)\, S_n(nt,0) \left[\gamma_\mu \frac{\not{n}}{2} \gamma_\nu\right] b(0)}$$

Wilson line $\mathcal{P} \exp\left[i g_s \int_0^t ds\; n\cdot A(ns)\right]$

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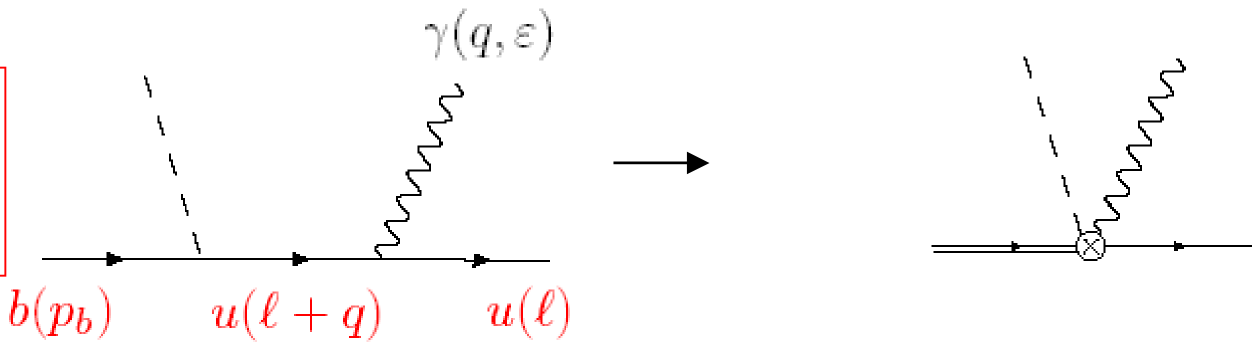

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$$\int d^4x e^{iq.x} T\{(\bar{q}\gamma_\nu P_L b)(0)\,, j_\mu^{\mathrm{e.m.}}(x)\} \longrightarrow \underbrace{1}_{\mathbf{H}} \cdot \int d\omega \underbrace{\left[\frac{i e_u}{\omega + i\epsilon}\right]}_{\mathbf{J}(\omega)} \underbrace{\int \frac{dt}{2\pi} e^{-i\omega t}\,\bar{u}(nt)\,S_n(nt,0)\left[\gamma_\mu \frac{\not{n}}{2}\gamma_\nu\right] b(0)}_{\mathbf{O}(\omega)}$$

Factorization II

Proof to all orders in α_s → Use **Soft-Collinear Effective Theory** (SCET)

Bauer, Fleming, Pirjol, Stewart; Beneke et al; Neubert-Hill

Same steps as in $B \to \ell\nu\gamma$ factorization

Sachrajda, Descotes-Genon
Lunghi, Pirjol, Wyler
Bosh, Hill, Lange, Neubert

$$\varepsilon^{*\mu} T_{\mu\nu}(q) \to -C_1^{(v)} e_u \int d\omega J(\omega) O_{1\nu}(\omega) - [C_2^{(v)} v_\nu + (C_1^{(v)} + C_3^{(v)}) \frac{n_\nu}{n\cdot v}] e_u \int d\omega J(\omega) O_2(\omega)$$

$$O_{1\mu}(\omega) = \int \frac{dt}{2\pi} e^{-it\omega}\, \bar{q}(nt) S_n(nt,0)\, \not{\varepsilon}_\perp^* \frac{\not{n}}{2} \gamma_\mu^\perp P_L\, b_v(0)$$

$$O_2(\omega) = \int \frac{dt}{2\pi} e^{-it\omega}\, \bar{q}(nt)\, S_n(nt,0) \not{\varepsilon}_\perp^* \frac{\not{n}}{2} P_R\, b_v(0)$$

$C_i^v(E_\gamma)$ and $J(\omega)$ are known to $O(\alpha_s)$

The soft matrix elements

- Above relation valid for any $B \to M_{\rm soft}\ell\nu\gamma$ (derived at operator level)

- If M=π can calculate matrix element using HHChPT, which incorporates heavy quark symmetry and chiral symmetry

Burdman, Donoghue
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Heavy mesons $\quad \bar{q}^a\, b \quad \leftrightarrow \quad H^a = \frac{1+\not v}{2}[B_a^{*\mu}\gamma_\mu - B_a\gamma_5]$

Goldstone modes $\quad \xi = e^{iM/f}$

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Chiral symmetry

$$\boxed{\begin{array}{c} L, R \in SU(3)_{L,R} \\ U(L,R,M) \in SU(3) \end{array}}$$

$$\xi \to L\xi U^\dagger = U\xi R^\dagger \qquad H_a \to H_b U^\dagger_{ba}$$

HQ spin symmetry

$$H \to S\, H$$

- Leading order chiral realization of bi-local operator ~ $(\bar{\mathbf{3}}_L, \mathbf{1}_R)$

$$O^a_\Gamma(\omega) = \int \frac{dt}{2\pi} e^{-it\omega}\, \bar{q}^a(nt) S_n(nt,0)\, P_R \Gamma\, b_v(0) \longrightarrow \frac{i}{4}\mathrm{Tr}\boxed{[\hat{\alpha}_L(\omega)}P_R\Gamma H_b \xi^\dagger_{ba}]$$

$$\boxed{\hat{\alpha}_L(\omega)} = a_1 + a_2 \not{n} + a_3 \not{v} + \frac{1}{2} a_4 [\not{n}, \not{v}]$$

$H \not{v} = -H$ → only two independent functions

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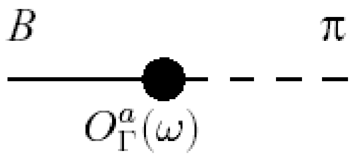
- Take B → vacuum matrix element + use definition of B-meson LCDA

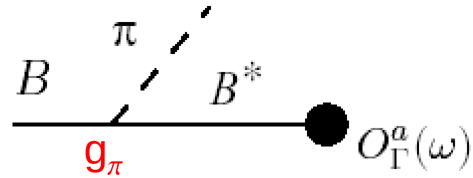
$$\boxed{\hat{\alpha}_L(\omega)} = -f_B \sqrt{m_B}[\not{\bar{n}} \phi_+(\omega) + \not{n} \phi_-(\omega)]$$

$$\int \frac{dt}{2\pi} e^{-i\omega t} \langle 0 | \bar{q}_i(nt) S_n(nt,0) b^j_v(0) | \bar{B}(v) \rangle = -\frac{i}{4} f_B m_B \left\{ \frac{1+\not{v}}{2} [\not{\bar{n}}\, n\cdot v\, \phi_+(\omega) + \not{n}\, \bar{n} \cdot v\, \phi_-(\omega)] \gamma_5 \right\}_{ji}$$

To LO in low energy expansion B → π matrix elements fixed in terms of B-meson LCDA !!

Factorization result





(similar results for A_i)

$$V_1 = 2e_u C_1^{(v)}(E_\gamma) \int d\omega J(\omega) S_1(\omega, p, p')$$

$$V_2 = \frac{e_u}{E_\gamma}\left(2C_1^{(v)}(E_\gamma) + C_2^{(v)}(E_\gamma) + 2C_3^{(v)}(E_\gamma)\right) \int d\omega J(\omega) S_2(\omega, p, p')$$

$$V_3 = \frac{2e_u}{n \cdot W} C_2^{(v)}(E_\gamma) \int d\omega J(\omega) S_2(\omega, p, p')$$

$$S_1(\omega, p, p') = -\frac{f_B m_B}{4 f_\pi} \phi_+^B(\omega) \left(1 + g_\pi \frac{(n - v) \cdot p'}{v \cdot p' + m_{B^*} - m_B}\right)$$

$$S_2(\omega, p, p') = \frac{g f_B m_B}{4 f_\pi} \phi_+^B(\omega) \frac{1}{v \cdot p' + m_{B^*} - m_B}$$

Non-perturbative B meson dynamics appears through the convolution:

$$V_i, A_i \propto \int d\omega\, J(\omega)\, \phi_+^B(\omega)$$

Phenomenology ($\bar{B}^0 \to \pi^+ e^- \bar{\nu}_e \gamma.$)

- Simplified analysis: neglect O(α_S) corrections to C_i and $J(\omega)$

- Input:

f_B	(200 ± 30) MeV	f_π	131 MeV
λ_B	(350 ± 150) MeV	g	0.5 ± 0.1
$\lvert V_{ub}\rvert$	0.004		

$$\lambda_B^{-1} = \int d\omega \, \frac{\phi_B^+(\omega)}{\omega}$$

induces largest uncertainty

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- BR output:

cuts	10^6 Br(fact)	10^6 Br(IB1)	10^6 Br(IB2)
$E_\gamma > 1$ GeV $E_\pi < 0.5$ GeV $\theta_{e\gamma} > 5^\circ$	1.2	2.4	2.8

Low's theorem extrapolated (PHOTOS)

Point-like B (Ginsberg)

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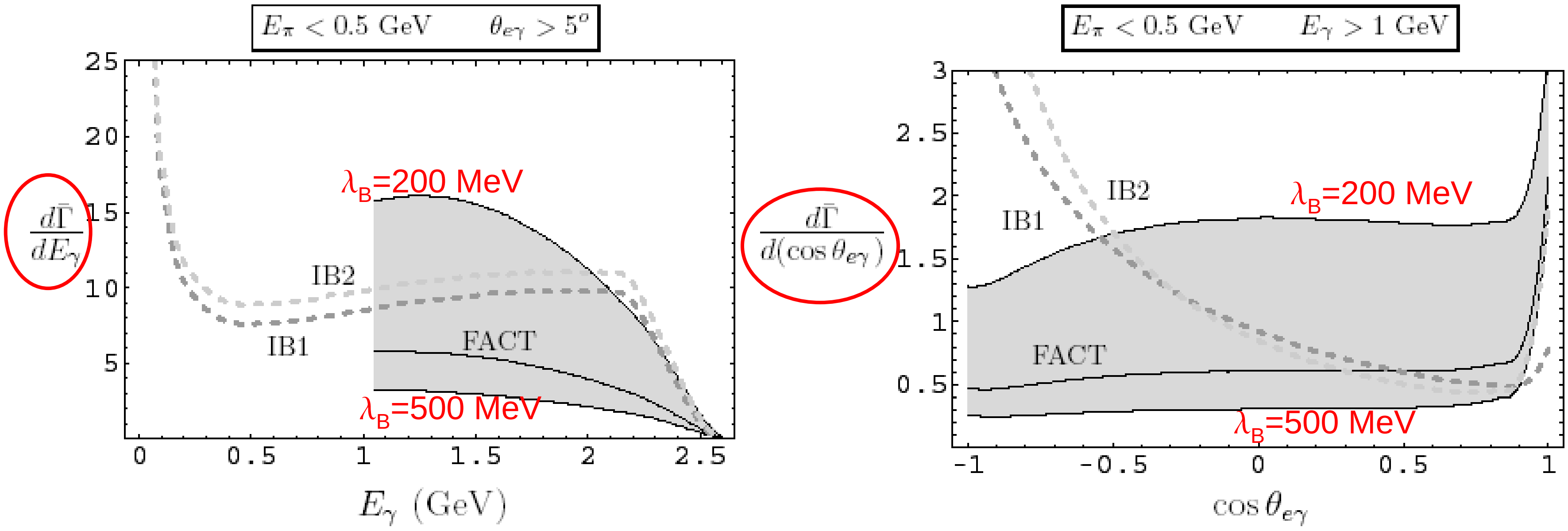
$$\mathrm{Br}^{\mathrm{cut}}_{\bar{B}\to\pi e\bar{\nu}\gamma}(\mathrm{fact}) = \left(1.2 \pm 0.2(g)\ {}^{+2.2}_{-0.6}(\lambda_B)\right) \times 10^{-6} \times \left(\frac{|V_{ub}|}{0.004}\right)^2 \times \left(\frac{f_B}{200\ \mathrm{MeV}}\right)^2$$

- Differential distributions in E_γ and $\cos(\theta_{e\gamma})$

$$\boxed{\bar{\Gamma} = 10^6\, \Gamma_{B^0}^{-1} \times \Gamma(\bar{B} \to \pi e \bar{\nu} \gamma)}$$



- **Factorization predictions are clearly distinct from IB1, IB2**
- BR + distributions show strong sensitivity to λ_B → in the future can obtain constraints on λ_B from this process (but need to assess size of chiral and $O(\alpha_S)$ corrections)

Conclusions

- Factorization for B → π lν γ in the region of hard γ and soft π
- Novelty: calculate soft matrix element in HHChPT

  B → π predicted in terms of B meson LCDA
- Phenomenology

  ➔ Factorization predictions for BR, spectrum, etc quite different from other methods on the market (PHOTOS, Ginsberg) → possibility to improve present Monte Carlo

  ➔ Strong sensitivity to λ_B → possibility to constrain it from B → π lν γ